Joint Aircraft Loading/Structure Response Statistics of Time to Service Crack Initiation

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A reliability analysis for predicting the statistical distribution of time to fatigue crack initiation for aircraft structures in service is presented. The present analysis utilizes the statistical data of the specimen fatigue tests, the full-scale structure tests, and the statistical dispersion of aircraft service loads. The statistical distribution of the time to fatigue crack initiation of the full-scale structure under laboratory loading spectrum is assumed to be Weibull. The service loads for gust turbulences are modeled as Poisson processes for transport-type aircraft, while the maneuver loads are modeled as compound Poisson processes for fighter and training aircraft. It is shown that the results of these models compare reasonably well with the available service data. A general formulation taking into account all the influencing statistical variables is also presented. It is demonstrated that the results of the present analyses have a good correlation with the available service data for time to crack initiation. It is found that the statistical distribution of time to fatigue crack initiation for aircraft structures in service is not Weibull and that the prediction on the basis of the Weibull distribution is unconservative, in particular in the early service time. The characteristics of the time to fatigue crack initiation for aircraft structures in service are discussed in detail.

Nomenclature

$f_{S(t)}(x)$	= probability density function of $S(t)$
$f_{N(t)}(x)$	= probability density function of $N(t)$
$F_T(t)$	= distribution function of time to first service crack initiation in a fleet of airplanes
$F_I(t)$	= distribution function of time T to service crack initiation
$F_I(t)$	= distribution function of time to first service crack initiation in a fleet of airplanes
$F_R(x)$	= distribution function of fatigue strength R (Eq. 1)
n	= total number of full-scale tests
$N(t)$	= number of gusts encountered by individual aircraft in t service-hours
N	= fleet size
$r(t)$	= fleet reliability in t service-hours
R	= fatigue strength; a random variable denoting the time to fatigue crack initiation of full-scale structures under laboratory test spectrum
$S(t)$	= number of equivalent laboratory flight-hours corresponding to service loads in t service flight-hours
s_f	= service scatter factor
t_r	= certified crack-free service lift
$\bar{T}_I$	= mean time to first crack initiation
$V_{N(t)}$	= coefficient of variation of $N(t)$, a measure of statistical dispersion (variability) of $N(t)$ [Eqs. (4) and (13)]
Y_j	= time to fatigue crack initiation of jth full-scale test
α	= shape parameter of Weibull distribution [Eq. (1)]
β	= scale parameter of Weibull distribution [Eq. (1)]
$\hat{\beta}$	= maximum likelihood estimator of β
γ	= parameter associated with maneuver loads
$\Gamma(x)$	= gamma function
λ	= parameter indicating statistical dispersion of gust loads [Eq. (4)]
$\bar{\lambda}$	= mean value of λ for maneuver loads, $\bar{\lambda}=\gamma/\theta$ [Eq. (9)]
ξ	= severity index
θ	= parameter associated with maneuver loads
$\mu_{N(t)}$	= mean value of $N(t)$
$\sigma_{N(t)}$	= standard deviation of $N(t)$

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I. Introduction

ONE ultimate goal in the design of aircraft structures is to insure a certain level of reliability for a fleet of airplanes during their service life. The time to fatigue crack initiation of aircraft structures in service plays an important role in determining their reliability. The time to crack initiation in service, however, is significantly influenced by many factors, such as a) fatigue strength (referred to as the time to crack initiation under laboratory spectrum loading), b) service loading, c) environment, d) manufacturing and inspection, e) design and stress analysis. Unfortunately, all these factors involve considerable statistical variability. The statistical variability of fatigue strength for components and full-scale structures under laboratory spectrum loading has been investigated. The random nature of aircraft loads resulting from atmospheric turbulence, maneuver, etc., has also been investigated. The environment, such as stress corrosion or corrosion fatigue, may have a significant effect on aircraft fatigue. The statistical variation of time to crack initiation for individual aircraft resulting from the manufacturing and assembling variability also influences fleet reliability in service. To account for these statistical uncertainties in a deterministic fashion in the preliminary fatigue design of aircraft structures, the use of several pertinent scatter factors was proposed by Crichlow.[1] Various concepts of the scatter factor approach have also been proposed in the past (e.g., in Refs. 2-8).

To predict the reliability of a fleet of airplanes in service and to correlate it with the in-service data, the statistical distribution of each influencing factor mentioned earlier must be characterized. Then, the reliability in service can be predicted through the analysis of these statistical values.

Recently, service data for crack initiation have become available for certain types of airplanes. Attempts have been made to correlate the service data with the predicted time to crack initiation on the basis of laboratory test results only.[9-11]

It is the purpose of this paper to perform an exploratory reliability analysis to predict the distribution of time to fatigue crack initiation for aircraft structures in service. The predicted result is then used for comparison with the available service data to verify the validity of the present analysis.

The reliability analysis approach employed herein is a quasi-static analysis, in the sense that the dynamic behavior of the aircraft structure has been reflected by the results of the laboratory full-scale tests. Loads experienced by each aircraft in service are modeled as Poisson-type random processes, indicating that there is no correlation for their occurrences. This quasi-static analysis, although not as accurate as the random vibration analysis,[12] is simple to perform and is useful in the preliminary design and analysis.

As mentioned previously, crack initiation in service is influenced by many statistical variables. Since, however, the statistical information regarding most of these variables is not available at this time, we shall assume that the variability of these factors is negligible and the main contributions to the service crack initiation are the fatigue strength and the service loading, where the fatigue strength is referred to as the time to crack initiation under laboratory spectrum loading. If this approach should result in poor correlation between the predicted results and the service data, we can then draw the conclusion that the environmental effect, the manufacturing variability, etc., are important factors for certain types of aircraft under certain type of mission operation, and an investigation regarding their effects is warranted.

A general formulation for the quasi-static analysis is presented in the Appendix. The formulation takes into account all the influencing factors, such as the fatigue strength, service loading, environmental effect, manufacturing and inspection, and design and analysis errors. The purpose of this formulation is to show that the analysis technique is available and it can be applied when the statistical information regarding all the influencing factors becomes available in the future.

The results of the present analysis have a good correlation with the available in-service data. It is indicated that the distribution of the time to crack initiation of aircraft structures in service is not Weibull, and that the prediction based on the Weibull distribution is unconservative in early service life. The present analysis enables one to obtain further insight into the distribution of the fatigue crack initiation of aircraft in service as well as the situation when the usage of the aircraft changes.

II. Prediction of Fatigue Strength of Aircraft Structures

The fatigue strength of material specimens has such a great dispersion that a probabilistic approach for fatigue strength prediction is essential. Hence, the establishment of a statistical distribution fot the fatigue strength (time to crack initiation under laboratory loading spectrum) R of aircraft structures is an important step toward the reliability prediction of aircraft structures under service loading. It is, however, impractical to establish such a statistical distribution through the extensive testing of full-scale structures, since full-scale testing is expensive and time consuming. In this connection, the attempt has been made to estimate the statistical distribution of time to crack initiation under laboratory spectrum loading using the following procedure[6-8]:

a) Assume that the distribution of fatigue strength (time to fatigue crack initiation under laboratory spectrum loading) of full-scale structures is a two-parameter Weibull distribution

$$F_R(x) = 1 - \exp\{-(x/\beta)^{\alpha}\} \tag{1}$$

in which $F_R(x)$ is the distribution function of R, which is the probability that the fatigue strength R is less than or equal to x. In Eq. (1), R is expressed in terms of x, the number of flight-hours under a laboratory fatigue test spectrum for the full-scale structure. α is the shape parameter and β is the scale parameter.

b) Assume that the fatigue strength of the coupon specimen is also Weibull distributed, and that the shape parameter α of the full-scale structure [Eq. (1)] can be approximated by the shape parameter of the coupon specimens. This assumption is based on the observation of extensive specimen test data that their statistical dispersion remains fairly constant and is not sensitive to the specimen geometry, testing method, testing machine, specimen size, etc. Compilation of coupon test data under constant amplitude and spectrum loading has been made and is available.[2,7,8,11]

c)The scale parameter β [Eq. (1)] is estimated from the result of the full-scale test using the maximum likelihood estimator[7]

$$\hat{\beta} = \left[\frac{1}{n}\sum_{j=1}^{n} Y_j^{\alpha}\right]^{1/\alpha} \tag{2}$$

where n is the number of full-scale tests, and Y_j is the result of the jth test. The number of full-scale tests n is usually very small. However, it has been plausibly shown by Freudenthal[13] that the effect of n on the scatter factor is not significant. As a result, the improvement in the estimation of β with the increase of n is also not significant.

It should be mentioned that the loading spectrum used for the full-scale test is an estimated spectrum representative of service loading. $\hat{\beta}$ is expressed in "number of laboratory flight-hours," being consistent with the argument x of the fatigue strength R[Eq. (2)]. The laboratory test spectrum (estimated load spectrum) may deviate significantly from the actual service spectrum, in particular, when the usage of the aircraft is changed. This problem will be discussed later.

III. Statistical Distribution of Service Load

It is well known that gust loads, maneuver loads, ground loads, and ground-air-ground loads are responsible for the fatigue damage of aircraft structures. The gust loads are random in nature.[14-17] Although the maneuver loads are controlled, they exhibit a considerable statistical variability (dispersion) depending on mission definition and flying technique (see Refs. 11, 18-20 for maneuver data). As a result, the maneuver loads have been treated as random (statistical) loads.[11,18-20] Such a premise will be employed in this paper. Although the ground-air-ground loads and ground loads are important for fatigue crack initiation, their statistical variabilities (dispersions) are smaller than that for gust loads or maneuver loads. Hence, the statistical variability (dispersion) ground-air-ground of the loads and the ground loads, although they may be important in some cases, will be neglected for the purpose of this paper. Since the statistical characteristics of gust loads and maneuver loads are significantly different, their statistical models will be discussed separately.

A. Gust Loads

According to available data, the occurrence of gust loads with intensity significant to fatigue damage can reasonably be assumed to be statistically independent. Based on this assumption, it can be shown[22] that the occurrence of gust loads follows a Piosson process. The assumption of statistical independence for the occurrence of gusts has been implied in Ref. 15, when all the turbulence patches are connected in series for the analysis of exceedance curves. Further application of such assumption has also been made in Ref. 16. Let $N(t)$ be the number of gusts (with intensity significant to fatigue damage) encountered by an individual aircraft of a fleet in t flight-hours of service. Then, the probability mass function of $N(t)$, which is the counter part of the density function for a continuous random variable, is given by the Poisson distribution

$$P[N(t)=n]=(\lambda t)^n e^{-\lambda t}/n! \tag{3}$$

where λ is the occurrence rate of gusts, which is assumed to be constant.

It follows from Eq. (3) that the mean value of $N(t)$, denoted by $\mu_{N(t)}$, is λt and the standard deviation, denoted by $\sigma_{N(t)}$, is $(\lambda t)^{1/2}$. Both the mean $\mu_{N(t)}$ and the standard deviation $\sigma_{N(t)}$ increase as the flight-hours t increase. The coefficient of variation, $V_{N(t)}$, of $N(t)$, which is the ratio of the standard deviation to the mean, is, therefore

$$V_{N(t)}=\sigma_{N(t)}/\mu_{N(t)}=1/(\lambda t)^{1/2} \tag{4}$$

Since the coefficient of variation is a measure of the statistical dispersion (variability) of $N(t)$, Eq. (4) indicates that the statistical dispersion of gust loading, encountered by aairplane in service, is inversely proportional to the square root of the service flight-hours t, i.e., the loading dispersion decreases as the flight-hours increase.

It should be mentioned that, not only the temporal occurrence of gust is random, as described by a Poisson process, but also the gust intensity σ is random. For instance, the intensity σ is considered a random variable following a half normal distribution in Ref. 15 (also in Ref. 12). Hence, for an overall assessment of gust and turbulence influences on structural fatigue, the variation in gust intensity must also be considered. The statistical variability of intensity σ, however, has been appropriately used in establishing the mean exceedance curves of gusts.[12,13,15] Therefore, we shall account for the statistical variability of gust intensity by utilizing the exceedance curves for the estimation of the overall gust variability. This will be discussed later in Sec. IV-C.

For the sake of simplicity in computation, it is convenient to approximate the discrete distribution of Eq. (3) by a continuous distribution, as follows

$$f_{N(t)}(x)=(\lambda t)^x e^{-\lambda t}/\Gamma(x+1);\quad 0<x<\infty \tag{5}$$

in which $\Gamma(\cdot)$ is the gamma function and $f_{N(t)}(x)$ is the density function of $N(t)$, which takes continuous values in $(0,\infty)$.

To evaluate the probability of crack initiation in service, the service loading should be converted into the laboratory spectrum in terms of the equivalent laboratory flight-hours, which is the argument of the fatigue strength R [see Eq. (2)]. The number of gusts $N(t)$ experienced by an aircraft in t service flight-hours is related to the number of (equivalent) laboratory flight-hours $S(t)$ through the following relationship

$$S(t)=N(t)/\lambda\xi \tag{6}$$

where $S(t)$ is a random variable representing the number of equivalent laboratory flight-hours corresponding to the random service load $N(t)$ in t service flight-hours. $\lambda\xi$ is the estimated (average) number of gust occurrences per flight-hour used in the laboratory test spectrum. Hence, the laboratory test spectrum represents the mean spectrum of the service load if $\xi=1$. ξ is an index denoting the severity of the laboratory test spectrum in relation to the actual service spectrum, and is called the severity index. This index is of particular importance when the usage of the aircraft deviates from its intended (design) usage.

The statistical distribution of the equivalent laboratory flight-hours $S(t)$, resulting from the statistical gust loading in service, can easily be obtained from Eq. (5) through the transformation of Eq. (6)

$$f_{S(t)}(x)=(\lambda t)^{x\lambda\xi}e^{-\lambda t}\lambda\xi/\Gamma(x\lambda\xi+1) \tag{7}$$

where $f_{S(t)}(x)$ is the probability density function of $S(t)$.

B. Maneuver Loads

For aircraft used in combat or training operations, such as fighters, the maneuver loading is important to fatigue damage. As mentioned previously, the maneuver loads are treated herein as random loads.[11,18-20] However, unlike the gust loading, the occurrence rate λ is no longer constant but a random variable. This is because, in any flight or training operation, the number of occurrences of maneuver loads per flight-hour varies significantly. Furthermore, important parameters, such as speed, altitude, and gross weight, which affect the induced stress level in the aircraft, also vary considerably. Hence, maneuver loading should be modeled as a compound Poisson process.

A special case of the compound Poisson process is that the distribution of the occurrence rate λ follows a gamma distribution

$$f_\lambda(y)=\theta^\gamma e^{-\theta y}y^{\gamma-1}/\Gamma(\gamma);\quad 0<\lambda<\infty \tag{8}$$

where $f_\lambda(y)$ is the probability density of λ, and θ and γ are the distribution parameters. The mean value of λ, denoted by $\bar{\lambda}$, can be obtained as

$$\bar{\lambda}=\gamma/\theta \tag{9}$$

Since the number of occurrences of the maneuver loading $N(t)$ in t service flight-hours is a Poission process with the occurrence rate λ, the conditional probability mass function of $N(t)$ given $\lambda=y$ is

$$P[N(t)=n\,|\,\lambda=y]=(yt)^n e^{-yt}/n! \tag{10}$$

The unconditional probability mass function of $N(t)$ is

$$P[N(t)=n]=\int_0^\infty P[N(t)=n\,|\,\lambda=y]f_\lambda(y)\,\mathrm{d}y \tag{11}$$

Substituting Eqs. (8) and (10) into Eq. (11), one obtains

$$P[N(t)=n]=\frac{\Gamma(n+\gamma)}{\Gamma(\gamma)\Gamma(n+1)}\left(\frac{\theta}{\theta+t}\right)^\gamma\left(\frac{t}{\theta+t}\right)^n \tag{12}$$

Equation (12) is recognized as the negative binomial distribution. The mean value $\mu_{N(t)}$, the standard deviation $\sigma_{N(t)}$, and the coefficient of variation $V_{N(t)}$, of $N(t)$ are given in the following

$$\mu_{N(t)}=\bar{\lambda}t \tag{13a}$$

$$\sigma_{N(t)}=(\bar{\lambda}t)^{1/2}\left(1+\frac{t}{\theta}\right)^{1/2} \tag{13b}$$

$$V_{N(t)}=(\bar{\lambda}t)^{-1/2}\left(1+\frac{t}{\theta}\right)^{1/2}=\left[\frac{1}{\bar{\lambda}t}+\frac{1}{\gamma}\right]^{1/2} \tag{13c}$$

where $\bar{\lambda}$ is given by Eq. (9).

It is observed from Eq. (13) that the statistical dispersion $V_{N(t)}$ of the maneuver loads, unlike that of gust loads, is not exactly inversely proportional to the square root of t, although it also decreases as the service flight-hours t increase. Equation (12) can be approximated by a continuous distribution

$$f_{N(t)}(x)=\frac{\Gamma(x+\gamma)}{\Gamma(\gamma)\Gamma(x+1)}\left(\frac{\theta}{\theta+t}\right)^\gamma\left(\frac{t}{\theta+t}\right)^x;\quad 0\le x<\infty \tag{14}$$

where $f_{N(t)}(x)$ is the density function of $N(t)$, which takes continuous values in $(0,\infty)$.

Now, the maneuver loads $N(t)$ experienced in t service flight-hours can be converted into the equivalent laboratory flight-hours $S(t)$ through

$$S(t) = N(t)/\bar{\lambda}\xi \tag{15}$$

This conversion is similar to that of Eq. (6). ξ again is a severity index indicating the relative severity of the laboratory test spectrum to the service spectrum. The probability density function of $S(t)$ can be obtained from Eq. (14) through the transformation of Eq. (15) as follows

$$f_{S(t)}(x) = \frac{\Gamma(\bar{\lambda}x\xi+\gamma)}{\Gamma(\gamma)\Gamma(\bar{\lambda}x\xi+1)}\left(\frac{\theta}{\theta+t}\right)^{\gamma}\left(\frac{t}{\theta+t}\right)^{\bar{\lambda}x\xi}\bar{\lambda}\xi \tag{16}$$

It should be noted that there are two distribution parameters involved in Eq. (16); i.e., θ and $\gamma(\bar{\lambda}=\gamma/\theta)$. These two parameters depend on the particular type of aircraft under the particular type of mission operation. They should be estimated using available service data, and will be discussed later.

The theoretical models for both the gust loads and the maneuver loads, established herein, indicate that the mean load $\mu_{N(t)}$ and the standard deviation $\sigma_{N(t)}$ increase as the service flight-hours t increase. The statistical dispersion (variability)

$$V_{N(t)} = \sigma_{N(t)}/\mu_{N(t)}$$

however, decreases as the service flight hours t increase. This trend is consistent with the service data,[11,15] and is also consistent with the intuition that the longer an airplane is flying the more likely that it will have been subjected to a variety of loadings and its total loadings will have averaged out.

IV. Fleet Reliability

A. Statistical Distribution of Time to Crack Initiation in Service

At some fatigue critical locations of an aircraft, crack initiation occurs in service before the service flight hour t when the fatigue strength R is exceeded by $S(t)$ [Eqs. (7) and (16)], where $S(t)$ represents the service loading converted into the equivalent laboratory flight-hours. Hence, the probability of crack initiation before t service flight hours, denoted by $F_T(t)$ is

$$F_T(t) = P[R \le S(t)] = \int_0^\infty F_R(x) f_{S(t)}(x)\,dx \tag{17}$$

in which $f_{S(t)}(x)\,dx$ is the probability that the service loading $S(t)$ (equivalent laboratory flight-hours) is in the interval $(x, x+dx)$, and $F_R(x)$ is the probability that the fatigue strength R is smaller than x. In Eq. (17) the distribution function $F_R(x)$ is given by Eq. (1), and the density function $f_{S(t)}(x)$ is given by Eqs. (7) and (16), for gust loading and maneuver loading, respectively. $F_T(t)$ is, thus, the distribution function for time T to crack initiation in service.

With the aid of Eqs. (1), (7), and (16), one obtains, for transport-type aircraft

$$F_T(t) = \int_0^\infty \{1 - e^{-(x/\lambda\beta\xi)^\alpha}\}\frac{e^{-\lambda t}(\lambda t)^x}{\Gamma(x+1)}\,dx \tag{18}$$

and for fighter (or training) aircraft

$$F_T(t) = \int_0^\infty \{1 - e^{-(x/\bar{\lambda}\beta\xi)^\alpha}\} \times \frac{\Gamma(x+\gamma)}{\Gamma(\gamma)\Gamma(x+1)}\left(\frac{t}{\theta+t}\right)^x\left(\frac{\theta}{\theta+t}\right)^\gamma dx \tag{19}$$

It can easily be observed that, if statistical dispersions $V_{N(t)}$ of both the gust loading and the maneuver loading are zero, the distribution of time to crack initiation T is Weibull.

Let t_r be the certified crack-free service life and s_f be a service scatter factor defined as

$$s_f = \beta/t_r \tag{20}$$

The distribution function of the crack-free service life can easily be expressed in terms of the scatter factor s_f through the transformation of Eq. (20).

B. Statistical Distribution of Time to First Crack Initiation

It has been pointed out (e.g., in Refs. 3-11), that a meaningful measure of safety for a fleet of N airplanes is the time to first crack initiation, which obviously is a random variable. The statistical distribution of time to first crack initiation in a fleet of N airplanes can be derived as follows:

The distribution function $F_T(t)$ for time T to crack initiation of one airplane under service loading has been derived in Eqs. (18) and (19). It is reasonable to assume that the event of crack initiation for each individual airplane in service is statistically independent. This is because the loading experienced by each individual airplane is statistically independent and the fatigue strength of each airplane can be assumed to be statistically independent. The distribution function of the time to first crack initiation in a fleet of size N, denoted by $F_I(t)$, can be written as

$$F_I(t) = 1 - [1 - F_T(t)]^N \tag{21}$$

and the fleet reliability $r(t)$, i.e., the probability of no crack initiation, is

$$r(t) = [1 - F_T(t)]^N \tag{22}$$

The mean time to first crack initiation, denoted by T_I, can be obtained from the distribution function $F_I(t)$ as follows

$$T_I = \int_0^\infty [1 - F_T(t)]^N dt = \int_0^\infty r(t)\,dt \tag{23}$$

Substituting Eqs. (18-20) into Eq. (22), one obtains

$$[r(s_f)]^{1/N} = 1 - \int_0^\infty \{1 - e^{-(x/\lambda\beta\xi)^\alpha}\}\frac{(\lambda\beta/s_f)^x e^{-(\lambda\beta/s_f)}}{\Gamma(x+1)}\,dx \tag{24}$$

for transport-type aircraft operation, and

$$[r(s_f)]^{1/N} = 1 - \int_0^\infty \{1 - e^{-(x/\bar{\lambda}\beta\xi)^\alpha}\} \times \frac{\Gamma(x+\gamma)}{\Gamma(\gamma)\Gamma(x+1)}\left[\frac{\beta/s_f}{\theta+(\beta/s_f)}\right]^x\left[\frac{\theta}{\theta+(\beta/s_f)}\right]^\gamma dx \tag{25}$$

for fighter aircraft and training operations.

Equations (24) and (25) express the relationship between the service scatter factor s_f, the fleet reliability $r(s_f)$, the fleet size N, the fatigue strength (α,β), and the service loading variabilityes (λ for gust load, and θ and γ for maneuver load).

C. Determination of Service Loading Parameters

In Eq. (4), the parameter λ, which represents the statistical dispersion of the gust loads, should be determined from gust data. Extensive turbulence data indicate[14] that the cumulative probability of exceedance for the gust velocity is significantly influenced by 1) locations of the flight, 2) altitude, 3) season, 4) time of day, 5) atmospheric stability, 6) terrain, 7) wind velocity, 8) wind angle with the aircraft, 9) vertical wind gradient, 10) storm or nonstorm weather, etc. A detailed classification for the effect of each factor is given in Ref. 14. Therefore, the exceedance of the gust velocity experienced by each individual airplane in a fleet exhibits a considerable statistical variation, as shown by figures in Ref. 14.

As mentioned previously, the statistical variability of the gust intensity σ has been accounted for in establishing the exponential form for exceedance curves of gust velocity (e.g., in Refs. 12, 14, 15). The parameter λ, which is related to the coefficient of variation $V_{N(t)}$ of gust occurrences [see Eq. (4)] should then be estimated from the statistical variability (dispersion) of the exceedance curves. Service data are available for the statistical variability of gust exceedance curves for particular usage of aircraft (e.g., in Ref. 14). A Poisson density function [Eq. (7)], normalized with respect to β [characteristic life of the fatigue strength, Eq. (1)] with $\lambda\beta = 20$ and $\xi = 1$, is given in Fig. 1 at the service flight-hours $t = (1/3)\ \beta$, $(2/3)\beta$, and β, respectively. The fact that the statistical dispersion of gust loads decreases as t increases is clearly demonstrated.

The same situation applies to maneuver loading. Data on VGH counts for various g levels of exceedance are available for some types of fighters (e.g., in Ref. 11), from which the parameters θ and γ ($\bar{\lambda} = \gamma/\theta$) should be determined. Service data on the dispersion $V_{N(t)}$ of 4g exceedance [see Eq. (13)] for F-4 fighters[11] at various service flight-hours t is shown in Fig. 2, in which $V^2_{N(t)}$ is plotted against $1/t$, where the abscissa $1/t$ is in 10^{-2} hr.$^{-1}$ For instance, the data point associated with 0.1 in the abscissa corresponds to service data measured at $t = 1000$ service flight-hr. It is observed from Fig. 2 that the service data can be fitted reasonably well by a straight line. This indicates that the compound Poisson model [see Eq. (13)] suggested herein for maneuver loading is plausible. From the fitter straight line of Fig. 2, it is estimated that $\gamma = 6.56$, $\theta = 180.2$, and $\bar{\lambda} = 0.0364$. With the aid of these estimated parameter values, Fig. 3 displays the distribution function of the negative binomial distribution [Eq. (16)], which is normalized with respect to its mean $\bar{\lambda}t$ for $\xi = 1.0$ and $t = 100$ service flight-hr. Also plotted in Fig. 3 as solid circles are the service data. It can be observed that the negative binomial distribution [Eq. (16)] fits the service data very well.

V. Correlation with Service Data

Consider the case of a transport-type aircraft with a shape parameter α of 5.27 for the fatigue strength R [Eq. (1)]. The distribution function for the time to crack initiation $F_T(t)$ is computed from Eq. (18) for $\xi = 1$ and various statistical dispersions λ of guest loading. The results are plotted in Fig. 4 in which the service flight-hours t are expressed in terms of the characteristic life β of the fatigue strength R [Eq. (1)] for cases $\lambda\beta = \infty$, $\lambda\beta = 60$, and $\lambda\beta = 6$, respectively. In Fig. 4, the solid curve represents the case where $\lambda\beta = \infty$, indicating zero statistical dispersion (variability) in service loading, and hence, it is the distribution of the fatigue strength $F_R(t)$ (Weibull distribution). The distribution of the first failure for a fleet of 1000 aircraft is also plotted in Fig. 4. Several interesting conclusions have been made from out analyses as follows:

1) The distribution function of the time to crack initiation in service $F_T(t)$ is no longer Weibull, although the distribution function of the fatigue strength R is Weibull. This can be observed from Eqs. (18) and (19) as well as Fig. 4. In-service data on time to crack initiation have been available for some types of aircraft (e.g., in Refs. 9 and 10), and attempts have been made to fit these data by use of the Weibull distribution. The results of this effort indicate that the Weibull distribution is frequently rejected in testing the hypothesis because of the low level of significance. This important conclusion is further confirmed by the service data, as will be shown later.

2) The statistical dispersion (variability) of time to crack initiation in service is larger than the statistical dispersion of the fatigue strength R (see Fig. 4). This originates from the fact that the fatigue strength R is statistically independent of the service loads encountered by each individual airplane. In fact, the square of the coefficient of variation of the time T to crack initiation in service is the sum of the square of the coefficients of variation of R and $S(t)$.

3) Because of higher statistical dispersion in service crack initiation, the probability of early crack initiation in service is higher than that predicted on the basis of the fatigue strength R alone. The use of the Weibull distribution for predicting the time to service crack initiation T is unconservative. This is clearly indicated by the distribution function of the first crack initiation in service given in Fig. 4, in which, for a 50% reliability, the first crack initiation for $\lambda\beta = 6$ is 0.08β, while it is 0.251β as predicted from the fatigue strength alone. This is also verified by the in-service data, as will be discussed later.

4) the median of the time to crack initiation is only slightly shifted from that of the fatigue strength (see Fig. 4) when the severity index $\xi = 1$. This provides a useful conclusion that the median life of the time to crack initiation in service can be approximated by the median of the fatigue strength R if the laboratory full-scale testing spectrum is the mean loading spectrum in service. The median of T, however, is shifted away from that of R if ξ is not equal to one and it is sensitive to ξ only.

5) It follows from the preceeding conclusions that the statistical dispersion of time to crack initiation T in service is sensitive to and influenced by α and λ only, both representing the statistical dispersions of the fatigue strength R and service loading $S(t)$, respectively. The median life of T is affected only by the severity index ξ. The lower tail of the distribution $F_T(t)$ is bent increasingly upward, with respect to that of the Weibull distribution $F_R(t)$, with increasing dispersion of $S(t)$, i.e., decreasing λ. The distribution $F_T(t)$, however, is shifted horizontally along the axis of service flight-hours t by ξ. As a result, λ and ξ provide plausible means and sufficient flexibility to bend the tail and to shift $F_T(t)$ along t so as to fit the service data very well.

Service data have been available for a fleet of 439, C-130 transport aircraft through the inspection for crack initiation. These data are analyzed statistically[9] for a center wing lower surface location. The statistical analysis takes into account both the cracked and the uncracked data observed during inspection to establish a statistical distribution. The statistical distribution for time to crack initiation in service is plotted in Fig. 5 as the solid curve. One laboratory full-scale test has been performed and the maximum likelihood estimator [Eq. (2)] is $\beta = 5640$ flight hrs.[9] With $\alpha = 4.0$,[6] the distribution of the fatigue strength $F_R(t)$, for time to crack initiation using Eq. (18) for $\xi = 1.0$, $\lambda\beta = 30$, is also plotted in Fig. 5 as circles. It is observed that a) the predicted distribution has an excellent correlation with that of the service data, in particular in the region of the lower tail, which is of primary concern to design engineers, and b) the prediction based on the Weibull distribution of the fatigue strength R along without accounting for the statistical variability of service loading is unconservative in the early service time. This further confirms our preceding conclusion.

Service data are also available for a fleet of 41 transport airplanes. The statistical distribution of time to crack initiation, resulting from the statistical analysis of service data, is plotted in Fig. 6 as a solid curve, for a location in the center wing butt line. To examine the validity of the Weibull approximation for the service data, a best-fit technique is employed to fit the service data distribution to a Weibull distribution. The result is $\alpha = 4$, $\beta = 1{,}693$ flight-hr, and it is plotted in Fig. 6 as a dashed curve. It can be observed from Fig. 6 that the prediction based on the best-fit Weibull distribution is unconservative in the region of the lower tail (early service time), which is of practical importance in design. With the aid of this

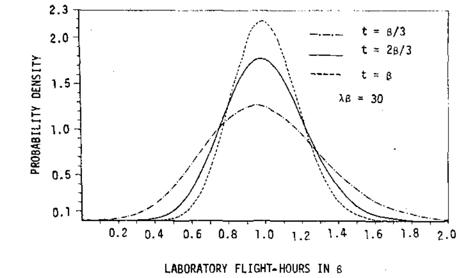

Fig. 1 Poisson density as function of service flight-hours.

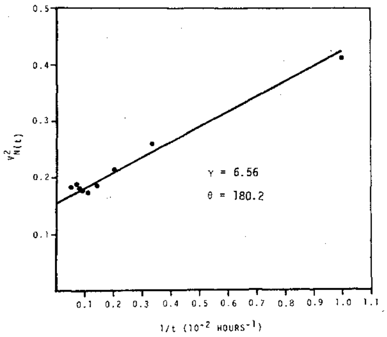

Fig. 2 Dispersion of 4g exceedance as function of inversed service flight-hours; maneuver loads.

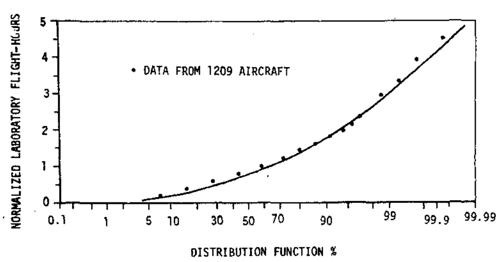

Fig. 3 Distribution function of exceedance for maneuver load.

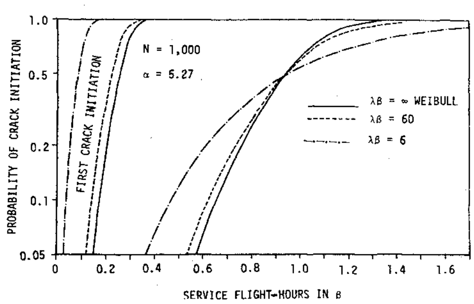

Fig. 4 Distribution functions of time to crack initiation in service.

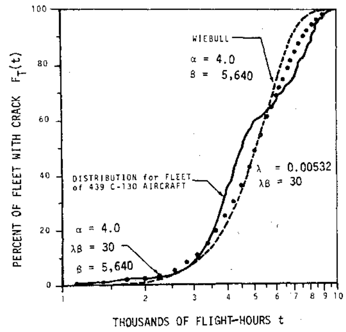

Fig. 5 Correlation between in-service data, Weibull distribution, and predicted distribution for time to crack initiation.

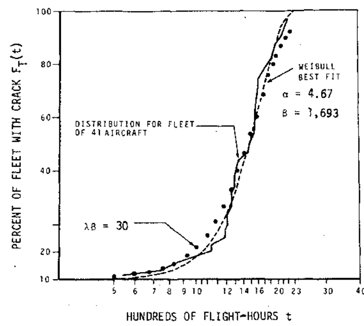

Fig. 6 Correlation between in-service data, best-fit Weibull distribution, and predicted distribution.

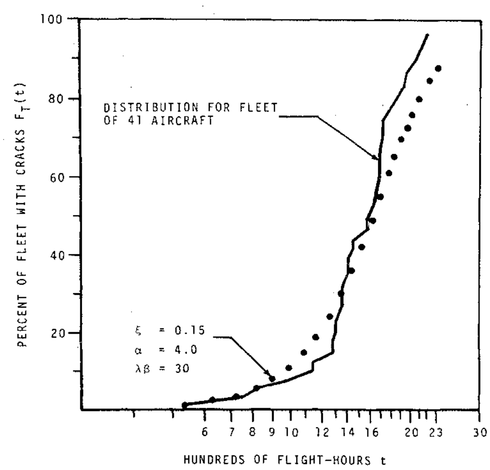

Fig. 7 Correlation between in-service data and predicted distribution.

Weibull distribution for $F_R(t)$, the predicted distribution in service $F_T(t)$ using Eq. (18), with $\lambda\beta=30$, $\xi=1.0$, is plotted in Fig. 6 as circles. It is demonstrated that the predicted distribution has better correlation with the service distribution in the region of early service time.

The maximum likelihood estimator β [Eq. (2)] from one full-scale test is 12,000 flight-hr. With this β and $\alpha=4.0$, the median of the fatigue strength R is far apart from that of the service data, indicating that the loading spectrum used for the full-scale test does not represent the mean service loading spectrum. As a result, a severity index of $\xi=0.15$ is introduced. Then, the prediction of $F_T(t)$ based on Eq. (18) for $\lambda\beta=30$ is plotted as cricles in Fig. 7. Again, the prediction has an excellent correlation with the distribution of service data, as indicated by Fig. 7. It is noted from Eq. (18) that the introduction of the severity index ξ is equivalent to shifting the characteristic life of the fatigue strength R from β to $\xi\beta$. The fact that $\xi=0.15$ indicates that the loading spectrum used for the laboratory full-scale test is unconservative. Likewise, ξ also serves as an index for the quality of the loading spectrum used in the full-scale test.

With the aid of Eq. (24), the reliability of a fleet of N airplanes is plotted in Fig. 8 as a function of the scatter factor s_f for $\alpha=5.27$, $\lambda\beta=30$, and $\xi=1.0$. The significant effect of the fleet size N on the fleet reliability is clearly demonstrated in Fig. 8.

It has been indicated that the negative binomial distribution fits the data of service loading for fighters very well. With the estimated parameter values $\gamma=6.56$, $\theta=180.2$ $(\bar{\lambda}=0.0364)$ (see Fig. 2), and the assumption that $\alpha=5.27$, the numerical results for the statistical distribution of the fatigue strength $F_R(x)$ and the predicted distribution of time to crack initiation $F_T(t)$ [Eq. (19)] have been examined. Note that the previous conclusions regarding the statistical distribution of time to crack initiation in service for transport-type aircraft also hold when applied to fighter aircraft. The reliability of a fleet of aircraft [Eq. (25)] is plotted as a function of the scatter factor s_f in Fig. 9 for various fleet sizes. Figure 9 clearly indicates the significant influence of the fleet size on the fleet reliability.

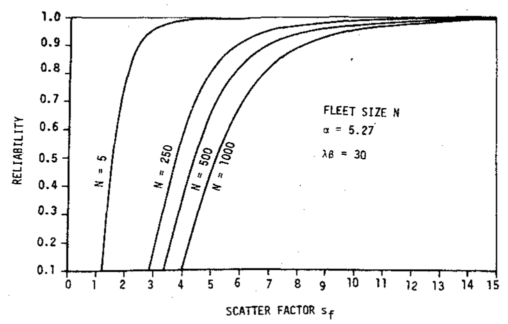

Fig. 8 Reliability as function of scatter factor and fleet size for transport-type aircraft.

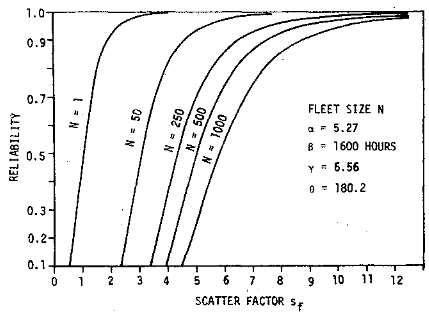

Fig. 9 Reliability as function of scatter factor and fleet size for fighter aircraft; $\beta = 1600$ laboratory flight-hr.

Currently available service data for fighters given in Ref. 11 are not sufficient to characterize the statistical distribution of time to crack initiation in service. This is because the service flight hours for the majority of airplanes are small and, hence, they do not have fatigue cracks yet. Therefore, no attempt is made herein to correlate the theoretical predictions with the service data. However, as service flight-hours of airplanes increase and more fatigue cracks are detected in the future, it would be possible to establish a statistical distribution of service data and to correlate it with the theoretical predictions.

VI. Conclusions and Discussion

A reliability analysis for predicting the time to crack initiation of aircraft structures in service is presented. The analysis utilized the statistical results of specimen fatigue tests, the full-scale structure tests, and the statistical dispersion of service loads. The statistical distribution of the fatigue strength under a laboratory loading spectrum is assumed to be Weibull. The service loads for gust turbulences are modeled as Poisson processes for transport-type aircraft, while the maneuver loads are modeled as compound Poisson processes for fighter aircraft. The statistical dispersion of both models for service loads decreases as the service time increases, being consistent with the observed data. It is shown that the results based on the use of models compare reasonably well with the service data. With the aid of available service data on time to crack initiation for some types of aircraft, it is shown that the prediction analysis presented herein has a good correlation with the service data. It is demonstrated that the prediction for time to crack initiation in service based on the Weibull distribution is unconservative in the early service time. The characteristics of the distribution for time to crack initiation in service are discussed in detail.

The available service data from inspection [9-11] are the number of flight-hours at which the crack at a particular location of the airplane is detected during inspection. These data, in effect, are the inspection data rather than data for time to crack initiation. The inspection data can be used to approximate the data for time to crack initiation only when the crack sizes detected during inspection do not differ significantly, or when the inspection is performed frequently enough, such as with the case of the C-130.

It is possible to convert inspection data into data for time to crack initiation. The procedure is as follows: Let a_0 be the crack size at which the crack initiation is defined. Suppose a crack with size $a_l > a_0$ is detected during inspection when the service flight-hours of the airplane are t_l. Using the crack propagation law in fracture mechanics (e.g., Ref. 12), one can calculate the time t_l^* for a crack with size a_0 to propagate to a size a_l. Then, the time to crack initiation for this airplane is $t_l - t_l^*$. In a similar manner, one can convert all the inspection data, $t_j (j=1,2,...)$ into data for time to crack initiation $t_j - t_j^* (j=1,2,...)$. The procedure is referred to herein as a "normalization procedure," in the sense that all the different crack sizes detected during inspection are normalized to a standard size a_0 at which the crack initiation is defined.

A very important parameter observed in the present analysis is the severity index ξ. A change in ξ will result in a shift in the characteristic life β of the fatigue strength and hence a shift for the predicted distribution for time to crack initiation in service. Note that the severity index ξ reflects the estimated loading spectrum used for the full-scale test in relation to the actual service loading spectrum. In other words, the analysis is critical with regard to how representative is the loading spectrum, which is used for the full-scale test. This is a rather difficult problem, since at the design and testing stage, no information regarding service loading is available and the loading spectrum used for the full-scale test is estimated on the basis of past experience as well as limited information for possibly similar aircraft in service. As a result, the prediction prior to service may not be accurate. It is, therefore, imperative to update the prediction, in particular ξ, as soon as the service data become available, for instance, data from the "lead the fleet aircraft." The Baysian approach may be used to update the prediction, however, further effort is needed in this regard.

Other parameters involved in the present analysis are the dispersions of both the gust loads and the maneuver loads, i.e., λ, γ, and θ. These parameters can be estimated from data on gust loads and maneuver loads, although effort is needed to characterize the statistical dispersion of both loads systematically. The effect of these parameters on the prediction analysis is to bend the tail of the distribution function for time to crack initiation and, hence, these parameters have a signficant effect on the prediction of first crack initiation.

In many circumstances, in particular for fighter and training aircraft operations, the subsequent usage (or mission characteristics) of the aircraft may be significantly different from the intended (design) usage under which the laboratory full-scale test is performed. Under this circumstance, the prediction analysis presented herein can be applied, except that the severity index ξ should be modified. The severity index ξ should be estimated from the relationship between the design load spectrum and the subsequent (changed) usage load spectrum. The relationship should be estimated through the analyses of load spectra for various usages and the application of the pertinent cumulative damage rule. Further research effort is needed in this area.

Finally, a general formulation accounting for all the statistical variables is presented in the Appendix. It is not used in the present analysis because of the lack of information regarding various statiatical variables. It is suggested that an effort be made to compile the statistical data for all the influencing variables so that more accurate predictions can be achieved through the analysis.

Appendix—General Formulation

In this formulation two basic random variables are considered: 1) the generalized fatigue strength R^*, and 2) the service load $S(t)$ within a service time interval $(0,t)$. The generalized fatigue strength R^* is a function of several random variables representing all the influencing factors, for instance, the fatigue strength of the aircraft structure R_1‡ the manufacturing and inspection R_2, the environmental effect R_3 (e.g., corrosion), the error in predicting the fatigue strength R_4, and the error in stress analysis R_5(see Ref. 1). Thus, the generalized fatigue strength R^* can be written as follows

$$R^* = g(R_1, R_2, \ldots R_n) \tag{A1}$$

in which R_1, R_2, ...,R_n are random variables representing all the possible influencing factors. The function g appearing in Eq. (A1) represents the interaction of various influencing factors.

Crack initiation occurs within the service time interval $(0,t)$ when the generalized fatigue strength R^* is exceeded by $S(t)$, i.e.

$$F(t) = P[R^* \leq S(t)] = \int_0^\infty F_{R^*}(x) f_{S(t)}(x)\,dx \tag{A2}$$

where $F(t)$ is the distribution function of crack initiation in the service time interval $(0,t)$. $F_{R^*}(x)$ is the distribution function of R^*, and $f_{S(t)}(x)$ is the probability density function of $S(t)$, given by Eq. (7) and (16).

In principle, the statistical distribution of the generalized strength R^* can be obtained from the statistical distributions of the influencing factors R_1, R_2,...R_n through the transformation of Eq. (A1). In reality, however, such a transformation may involve serious mathematical difficulty. Based on the physical reasoning and the extreme value statistics, it may be reasonable to assume that the statistical distribution of the generalized fatigue strength R^* is Weibull. Accordingly, the remaining task is to evaluate the mean value μ_{R^*} and the standard deviation σ_{R^*} from the statistical distribution of R_1, R_2,...R_n. It follows from Eq. (A1) that

$$\mu_{R^*} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, x_2, \ldots x_n) \times \left[\prod_{j=1}^{n} F_{R_j}(x_j) \right] dx_1 dx_2 \ldots dx_n \tag{A3}$$

$$\sigma_{R^*}^2 = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g^2(x_1, x_2, \ldots x_n) \times \left[\prod_{j=1}^{n} F_{R_j}(x_j) \right] dx_1 dx_2 \ldots dx_n - \mu_{R^*}^2 \tag{A4}$$

in which it has been assumed that R_1, R_2...R_n are statistically independent, and $F_{R_j}(x_j)$ is the probability density of R_j.

Since the statistical distribution of R^* is assumed to be Weibull, the shape parameter α^* and the scale parameter β^* can be obtained from μ_{R^*} and σ_{R^*} [Eqs. (A3) and (A4) using the following two equations

$$\left[\Gamma\left(\frac{2}{\alpha^*}+1\right) - \Gamma^2\left(\frac{1}{\alpha^*}+1\right) \right]^{1/2} / \Gamma\left(\frac{1}{\alpha^*}+1\right) = \sigma_{R^*}/\mu_{R^*} \tag{A5}$$

$$\beta^* = \mu_{R^*}/\Gamma\left(\frac{1}{\alpha^*}+1\right) \tag{A6}$$

Therefore, the statistical distribution of time to crack initiation in service is given by Eq. (18) for transport aircraft and by Eq. (19) for fighter aircraft, where α and β should be replaced by α^* and β^*, respectively.

‡For simplicity in further development, R_1 is used herein, which is the same as R used in the text.

Depending on the functional form of $g(R_1, R_2, \ldots, R_n)$ it may not be possible to carry out the multiple integration given by Eqs. (A3) and (A4), either analytically or numerically. An alternate approach using the method of perturbation[21] for the evaluation of μ_{R^*} and σ_{R^*} is suggested herein.

Let μ_{kj} be the kth central moment of R_j. It is realized that μ_{1j} and μ_{2j} are the mean and the variance of R_j, respectively. The function $g(x_1, x_2, \ldots x_n)$ is first expanded about $(\mu_{11}, \mu_{12}, \ldots, \mu_{1n})$, the point at which each of the random variables takes its mean value, into a multivariate Taylor series. The mean value μ_{R^*} is obtained by taking the expectation of the Taylor series

$$\mu_{R^*} \simeq \bar{g} + \frac{1}{2} \sum_{j=1}^{n} \frac{\partial^2 \bar{g}}{\partial x_j^2} \mu_{2j} + \ldots \tag{A7}$$

in which $\bar{g}$ and $\partial^2 \bar{g}/\partial x_j^2$ denote g and $\partial^2 g/\partial x_j^2$, respectively, evaluated at μ_{1j} $(j=1,2,\ldots n)$, i.e., with $(x_1, x_2, \ldots, x_n)$ substituted by $(\mu_{11}, \mu_{12}, \ldots \mu_{1n})$. In Eq. (A7), higher order terms have been truncated. Using the Taylor series of $g(x_1, x_2, \ldots, x_n)$ in a similar manner, one obtains σ_{R^*} with the truncation of higher order terms as follows

$$\sigma_{R^*}^2 \simeq \sum_{j=1}^{n} \left(\frac{\partial \bar{g}}{\partial x_j} \right)^2 \mu_{2j} + \sum_{j=1}^{n} \left(\frac{\partial \bar{g}}{\partial x_j} \right) \left(\frac{\partial^2 \bar{g}}{\partial x_j^2} \right) \mu_{3j} + \ldots \tag{A8}$$

Frequently, it is a satisfactory approximation to neglect the second term on the right-hand side of Eq. (A8)

$$\sigma_{R^*}^2 \simeq \sum_{j=1}^{n} \left(\frac{\partial \bar{g}}{\partial x_j} \right)^2 \mu_{2j} \tag{A9}$$

It is observed from Eqs. (A7) and (A9) that, if the method of perturbation is used for evaluating μ_{R^*} and σ_{R^*} the probability density function $f_{R_j}(x_j)$ $(j=1, 2, \ldots n)$ need not be established for each influencing factor R_j. The statistical information required for each influencing factor is the mean μ_{1j} and variance μ_{2j} $(j=1,2,\ldots n)$ only.

Finally, if the statistical dispersions of other influencing factors R_j $(j=2,3,\ldots n)$ except R_1, are negligible, then $R^* = R_1$. Hence the general formulation degenerates into the special case where $\alpha^* = \alpha$ and $\beta^* = \beta$. This special case has been discussed in detail in the text.

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